Numerical Modelling of Surface Forest Fire Spread in Nonuniform Woodland

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Abstract. The problem of computer modeling of two-dimensional surface forest fire spread is considered. The article describes the initial-boundary value problem in a form of a system of partial differential equations. Presented mathematical model of surface forest fires spread calculates the evolution of the temperature, of combustible forest materials, volume fractions of the multiphase reactive medium, mass concentration of components of a gas phase. The system of PDEs is solved numerically. Authors explicit numerical schemes with a uniform grid over coordinates and variable steps over time are used. Time steps are selected in accordance with stability conditions of the numerical scheme, and with the speeds of physical and chemical processes at each specific step. Examples illustrate the results of numerical experiments showing how fire front spreads in different directions in windy conditions. Qualitative differences in the geometry and dynamics of temperature density maps are shown for a range of fuelbreak sizes.

Keywords: surface forest fire, physical and chemical processes of combustion, computer model, numerical method, heat and mass transfer equations, temperature density map, Wolfram Mathematica, fuelbreak.

1 Introduction

Of the many natural and anthropogenic factors affecting the state and dynamics of forest ecosystems, fires are dominant and cause significant material and environmental damage. Forest fires are divided into underground fires, surface fires, passive crown fires, active crown fires (dependant from surface fire), running crown fires (independent from surface fires), and mass fires. The most common type of forest fire is a surface fire. It involves materials such as dry grass, fallen leaves and needles lying on the ground, shrubs and small hardwood trees. Surface fire is generally slow moving and flames can rise almost one to two meters high. As the surface fire intensifies by burning more material, when heavier bushes and medium size trees start burning then the flames may rise as high as five meters or more.

Problems associated with forest fires are important for many countries due to the fact that monitoring and management of forest fires require significant material costs. Therefore, the current task is to ensure the effectiveness of forest fire services with the help of mathematical methods and modern information technologies. The
development of appropriate computer models and software, their inclusion in the
decision support systems for the prevention of emergency situations in forests and
surrounding areas is required for reasonable, successful actions for the prevention,
elimination of forest fires. The development of forest fires mathematical models
started from 1940s but many problems have not been yet resolved. In accordance
with scientific publications [1] in this article we use the following classification of
computer models of forest fires: empirical (statistical), semi-empirical, and theoretical
(mathematical).

Empirical (statistical) models calculate the rate of forest fire spread by searching
for historical information of statistical correlation between fire propagation speed
and some controlled parameters. Such an approach does not study the mechanism
of the phenomenon; the results, strictly speaking, cannot be extended beyond the
applicability of the statistics used, and even within them such a forecast is made only
with a certain probability. The issues of development and examples of application
of such models can be found in [3, 4].

In semi-empirical models general laws (conservation of energy, mass and amount
of motion) are used to determine the characteristics of fire propagation. They are
written in the form of simplified dependencies, and the corresponding coefficients
are selected by generalizing the experimental information. Semi-empirical models
are more adequate in comparison with empirical (statistical) models. Development
and generalization of traditional models of this type are proposed in [3, 4]. Semi-
empirical models are adequate in situations similar to those in which experimental
data were collected. Such models are much easier to verify than theoretical ones.
Software realization of the proposed computer models of the above types, the tech-
nical aspects of their development and capabilities are described in [5].

Theoretical models are based on the laws of continuum mechanics, other funda-
mental laws of physics and chemistry. Only they describe the processes in dynamics
taking into account general and local factors. This type of model can answer a very
wide range of questions and solve problems for real cases with nonuniform distribu-
tion of parameters. Mathematical descriptions of theoretical models are given, as a
rule, in the form of a large system of partial differential equations.

The development of theoretical models, the proposed numerical method for solv-
ing the corresponding boundary value problems for modelling of crown fires can be
found in articles [6, 7]. Representative results of simulation of crown forest fires
are given in articles [8, 9, 10]. Examples of typical fire propagation scenarios are
considered. The above-mentioned papers illustrate the use of developed computer
model to estimate effects of several typical fire fighting activities.

2 Mathematical model of surface forest fires spread

Basic concepts. Presented mathematical model of forest fires spread calculates the
evolution of the following values: $T$—temperature (in Kelvins) of combustible forest
materials (CFM); $\varphi_j$, $j = 1, 2, 3, 4$—volume fractions of the multiphase reactive
medium, where $\varphi_1$ corresponds to dry organic substance, $\varphi_2$—water in liquid-drop
state combined with CFM, \( \varphi_3 \)—coke (condensed pyrolysis product), \( \varphi_4 \)—mineral part of CFM (ash); \( c_\nu, \ \nu = 1, 2, 3 \)—mass concentration of components of a gas phase, where \( c_1 \) corresponds to oxygen (O\(_2\)), \( c_2 \)—to combustible gases (combustible pyrolysis product components), \( c_3 \)—mixes of other gases (inert components of air, water vapor, inert products of reactions of pyrolysis, coke burning and of combustible gases oxidation).

Among physical and chemical processes of CFM burning, the following are defined: the heat supply due to convection, heat conductivity and radiation, forest fuel heating, drying and pyrolysis, burning of gaseous, disperse and solid products of pyrolysis. The derivation of the equations, the model justification, the numerical scheme and the organization of calculations are given in the publications [6, 7, 11].

Relatively to unknown functions \( \varphi_j (j = 1, 2, 3, 4), c_\nu (\nu = 1, 2, 3) \) and \( T \), which depend on time and spatial coordinates, we formulate the initial-boundary value problem in the form of the following system of PDEs:

\[
\frac{\partial \varphi_1}{\partial t} = \Phi_{\varphi_1}(\varphi_1, T), \quad \frac{\partial \varphi_2}{\partial t} = \Phi_{\varphi_2}(\varphi_2, T), \quad \frac{\partial \varphi_3}{\partial t} = \Phi_{\varphi_3}(\varphi_1, \varphi_3, c_1, c_2, T), \quad \frac{\partial \varphi_4}{\partial t} = 0, \tag{1}
\]

\[
\frac{\partial c_1}{\partial t} + (V, gradc_1) - \frac{1}{\rho_5} div(\rho_5 D_T gradc_1) = \Phi_{c_1}(\varphi_1, \varphi_2, \varphi_3, c_1, c_2, T), \tag{2}
\]

\[
\frac{\partial c_2}{\partial t} + (V, gradc_2) - \frac{1}{\rho_5} div(\rho_5 D_T gradc_2) = \Phi_{c_2}(\varphi_1, \varphi_2, \varphi_3, c_1, c_2, T), \tag{3}
\]

\[
\frac{\partial T}{\partial t} + \rho_5 c_{p_5} (V, gradT) - div(\lambda_T gradT) = \Phi_T(\varphi_1, \varphi_2, \varphi_3, c_1, c_2, T), \tag{4}
\]

\[
\rho_5 c_{p_5} + \sum_{j=1}^{4} \rho_j c_f c_{pj} \tag{5}
\]

The defining functions are:

\[
\Phi_{\varphi_1}(\varphi_1, T) = -\frac{R_1}{\rho_1}, \quad \Phi_{\varphi_2}(\varphi_2, T) = -\frac{R_2}{\rho_2}, \tag{6}
\]

\[
\Phi_{\varphi_3}(\varphi_1, \varphi_3, c_1, c_2, T) = \frac{\alpha c_1 R_1}{\rho_3} - \frac{M_C R_3}{M_1 \rho_3}, \tag{7}
\]

\[
\Phi_{c_1}(\varphi_1, \varphi_2, \varphi_3, c_1, c_2, T) = \frac{1}{\rho_5} \left( R_{51} - c_1 Q - \frac{\alpha}{c_{p_5} \Delta h} (c_1 - c_{1\infty}) \right), \tag{8}
\]

\[
\Phi_{c_2}(\varphi_1, \varphi_2, \varphi_3, c_1, c_2, T) = \frac{1}{\rho_5} \left( R_{52} - c_2 Q - \frac{\alpha}{c_{p_5} \Delta h} (c_2 - c_{2\infty}) \right), \tag{9}
\]

\[
\Phi_T(\varphi_1, \varphi_2, \varphi_3, c_1, c_2, T) = \frac{q_0 R_5 - q_3 R_2 + q_3 R_3 - \frac{\alpha}{\lambda_T} (T - T_\infty) - 4\kappa R \sigma T^4}{\rho_5 c_{p_5} + \sum_{j=1}^{4} \rho_j c_f c_{pj}}, \tag{10}
\]

\[
\sum_{\nu=1}^{3} c_\nu = 1, \quad \rho_5 = \frac{\rho_\infty T_\infty}{M_\infty T} \left( \sum_{\nu=1}^{3} \frac{c_\nu}{M_\nu} \right)^{-1}, \quad Q = (1 - \alpha_c) R_1 + R_2 + \frac{M_C}{M_1} R_3. \tag{11}
\]
\[ R_1 = k_{01} \rho_1 \varphi_1 \exp\left(-\frac{E_1}{RT}\right), \quad R_2 = k_{02} T^{-1/2} \rho_2 \varphi_2 \exp\left(-\frac{E_2}{RT}\right), \quad (12) \]

\[ R_3 = k_{03} s_c \varphi_3 \rho_3 c_1 \exp\left(-\frac{E_3}{RT}\right), \quad (13) \]

\[ R_{51} = -R_3 - \frac{R_5 M_1}{2M_2}, \quad R_{52} = (1 - \alpha_c) \nu_G R_1 - R_5, \quad (14) \]

\[ R_5 = \rho_5 \min(c_2, \frac{M_2}{2M_1} c_1) k_{CO} \exp\left(-\frac{E_{CO}}{RT}\right). \quad (15) \]

Here \( t \) is the time; \( V \) is equilibrium wind speed vector; \( T_\infty \) is unperturbed ambient temperature; \( \rho_j, j = 1, 2, 3, 4 \) is the \( j^{th} \) phase density; \( \rho_5 \) is the density of a gas phase (a mix of gases); \( \rho_\infty \) is unperturbed density of a mix of gases (air density); \( c_{1\infty} \) and \( c_{2\infty} \) are mass concentrations of oxygen and combustible gases in unperturbed atmosphere; \( M_\nu, \nu = 1, 2, 3 \) are molecular masses of gas phase components; \( M_C \) is a molecular mass of carbon, \( M_\infty \) is a molecular mass of air; \( c_{\varphi j}, j = 1, 2, 3, 4 \) is the \( j^{th} \) phase thermal capacity; \( c_{\varphi 5} \) is thermal capacity of a gas phase; \( Q \) is a mass rate of generation gas phase; \( \lambda_T \) is turbulent thermal conductivity; \( D_T \) is the diffusion coefficient; \( q_2, q_3 \) and \( q_5 \) are heat effects of processes of evaporation, burning of the condensed fuel and of gaseous combustible pyrolysis products accordingly; \( \Delta h \) is a height of forest fuel (CFM); \( \alpha \) is a coefficient of heat exchange between the atmosphere and a forest canopy; \( \kappa_R \) defines integrated absorption coefficient; \( \sigma \) is the Stefan-Boltzmann constant; \( R_1 \) is a mass rate of reaction of dry CFM pyrolysis (chemical decomposition of substance by heating with allocation of combustible gases), \( R_2 \) is a mass rate of reaction of moisture evaporation from CFM (drying), \( R_3 \) is a mass rate of reaction of coke burning; \( R_{51}, R_{52} \) are mass rates of generation (disappearance) of oxygen, combustible gases; \( R_5 \) is mass rate of reaction of burning (oxidation) of combustible gases; \( k_{01}, k_{02}, k_{03}, k_{CO} \) are pre-exponential factors of chemical reactions, \( E_1, E_2, E_3, E_{CO} \) are the activation energies of chemical reactions, \( R \) is universal gas constant, \( s_\nu \) defines specific surface of the condensed product of pyrolysis (of coke), \( \alpha_c \) is the coke value of CFM, \( \nu_G \) is a proportion of gaseous combustible pyrolysis products.

### 3 Initial and boundary conditions

*Initial and boundary conditions.* Initial distributions of volume fractions \( \varphi_j \), of mass concentrations \( c_\nu \) and of temperature \( T \) are defined in a whole domain of a solution. Let's define the domain as \( G \), and its boundary as \( B \). The domain \( G \) can be divided by three subdomains \( G = G_{fire} \cup G_- \cup G_+ \). These subdomains are not necessarily simply connected, their mutual geometry can be quite complex, for example, for multiple spot fires. \( G_- \) and \( G_+ \) indicate burnt and unburned areas removed from fire zone to adequate distance (these subdomains are characterized by unperturbed values \( c_{1\infty}, c_{2\infty} \) and \( T_\infty \)). In fire area \( G_{fire} \) initial distributions of values \( \varphi_j(j = 1, 2, 3, 4) \), \( c_\nu(\nu = 1, 2, 3) \) and \( T \) must be "self-consistent", because there are certain physical connections between all of these values that must be taken
into account. These issues are rather complex and require separate discussion [7]. Initial distributions in burnt \( G_- \) and unburnt \( G_+ \) areas are given as follows:

\[
T|_{t=0}(G_- \cup G_+) = T_\infty, \quad (16)
\]
\[
c_1|_{t=0}(G_- \cup G_+) = c_{1\infty}, \quad c_2|_{t=0}(G_- \cup G_+) = c_{2\infty}, \quad (17)
\]
\[
c_3|_{t=0}(G_- \cup G_+) = 1 - c_{1\infty} - c_{2\infty}, \quad (18)
\]
\[
\varphi_1|_{t=0}(G_+) = \frac{\rho_0}{\rho_1}, \quad \varphi_2|_{t=0}(G_+) = (1 - \zeta)W\frac{\rho_0}{\rho_2}, \quad (19)
\]
\[
\varphi_3|_{t=0}(G_+) = 0, \quad \varphi_4 = 0 \quad (20)
\]
\[
\varphi_1|_{t=0}(G_-) = 0, \quad \varphi_2|_{t=0}(G_-) = 0, \quad \varphi_3|_{t=0}(G_-) = \alpha_e \frac{\rho_0}{\rho_3}. \quad (21)
\]

It is supposed that in \( G_- \) area forest fuel is fully burnt; the bulk densities \( \rho_0 \) of typical layer of CFM, phase densities \( \rho_j (j = 1, 2, 3, 4) \), \( W \) moisture content of forest fuel and ash content \( \zeta \) of combustible forest materials can be heterogeneous. All these distributions in the area of modeling must be provided as functions of coordinates. At the boundary \( B \) of the modeling domain \( G \) we define “weak” boundary conditions:

\[
\frac{\partial T}{\partial n}|_B = 0, \quad \frac{\partial c_1}{\partial n}|_B = 0, \quad \frac{\partial c_2}{\partial n}|_B = 0. \quad (22)
\]

4 The results of fires modeling in forests with fuelbreaks

The system of PDEs described above was solved numerically using the Wolfram Mathematica system [12, 13]. We used authors explicit numerical schemes with a uniform grid over coordinates and variable steps over time. Time steps were chosen in accordance with stability conditions of the numerical scheme, and with the speeds of physical and chemical processes at each specific step [7].

The strongest change in environmental condition parameters occurs in the zone called fire front, which propagates at some speed by the territory, covered with forest, and is visually observed as a zone captured by a flame. As a rule the effects of smoke generation over a big territory and clouds formation over a fire zone, because of condensation of the water vapor released during combustible forest materials burning, take place. We attempt here to describe the various key components that characterize fuelbreaks, evaluate their use, and discuss alternatives to traditional fuelbreak approaches. A fuelbreak is "a strategically located wide block, or strip, on which a cover of dense, heavy, or flammable vegetation has been permanently changed to one of lower fuel volume or reduced flammability” [14]. The effectiveness of fuelbreaks remains a subject of debate within and outside of the fire management community. There are many reasons for this broad range of opinion, among them that objectives can vary widely, fuelbreak prescriptions (width, amount of fuel reduction, maintenance standards) may also vary, they can be placed in many different

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fuel conditions, and may be approached by wildland fires under a variety of normal to extreme weather conditions.

Let’s consider the square area of the forest. The “burning” starts at the origin of the coordinate system. For simplicity, the direction of the wind in the forest canopy is along the axis Ox (from left to right on the graphs given). In this case, in the direction of the wind, against the wind and perpendicularly (on one of the flanks) there are areas with no combustible vegetation (glades). Graphical visualization of the described configuration and written expressions are given on figures 1–3.

![Graphs](image)

Figure 1: Glades size is 0.5 m. Fire gets round the glades

Following values are used for the modeling: \( V = 1.5 \text{ m/s}, W = 10\%, \rho_0 = 5 \text{ kg/m}^3, \Delta h = 0.1 \text{ m}, T_\infty = 300 \text{ K}, \rho_1 = 500 \text{ kg/m}^3, \rho_2 = 1000 \text{ kg/m}^3, \rho_3 = 200 \text{ kg/m}^3, \rho_4 = 200 \text{ kg/m}^3, \rho_\infty = 1.15 \text{ kg/m}^3, c_{1\infty} = 0.23, c_{2\infty} = 0, M_1 = 32, M_2 = 28, M_3 = 29, M_\infty = 12, M_\infty = 29, c_{p1} = 2000 \text{ J/(kg \cdot K)}, c_{p2} = 4180 \text{ J/(kg \cdot K)}, c_{p3} = 900 \text{ J/(kg \cdot K)}, c_{p4} = 1000 \text{ J/(kg \cdot K)}, c_{p5} = 1000 \text{ J/(kg \cdot K)}, q_2 = 3 \cdot 10^6 \text{ J/m}^2 \cdot \text{s} \)
\[ J/kg, \quad q_3 = 1.2 \cdot 10^7 \quad J/kg, \quad q_3 = 10^7 \quad J/kg, \quad \lambda_T = 1000 J/(m \cdot s \cdot K), \quad D_T = 1.5 m^2/s, \]
\[ \alpha = 100 \quad W/(m^2 \cdot K), \quad \alpha_c = 0.1, \quad \nu_G = 0.8, \quad \kappa_R = 1.5 \quad m^{-1}, \quad S_{\sigma} = 1000 \quad m^{-1}, \quad E_1/R = 9400 \quad K, \quad E_2/R = 6000 \quad K, \quad E_3/R = 10000 \quad K, \quad E_{CO}/R = 11649 \quad K, \quad k_{01} = 3.63 \cdot 10^4 \quad s^{-1}, \]
\[ k_{02} = 6 \cdot 10^5 \quad K^{0.5} \quad s^{-1}, \quad k_{03} = 1000 \quad s^{-1}, \quad k_{CO} = 7.05 \cdot 10^6. \]

Figure 2: Glades size is 1.0 m. Fire gets round the glades

Figures 1 3 show the temperature distribution maps at different points of time: during the "rounding" of the glades, when the fronts are open, and after the glades are passed through by fire front. The results of three variants of modeling are presented. They, differ only by the sizes of square glades: 0.5 meters, 1.0 meters and 1.5 meters wide, respectively. The fire spreads like an ellipse, reaches the glades and begins to curve around them. The results show that three variants of the further development of the process are possible: all fire fronts close again independently of the direction of the wind (figure 1); when fuelbreak width increases to 1 meter then one front of fire, which is opposite to the wind direction, doesn’t close (figure 2);
when the glades sizes are large enough then they cause fire to stop in all directions except the direction of the wind. The last graph at figure 3 shows interesting fire dynamics. The fire front began to move along the glade in the direction opposite to wind direction.

Figure 3: Glades size is 1.5 m. Fire gets round the glades

There is a clear theoretical basis for concluding that fuelbreaks will alter fire behavior in ways amenable to limiting both the sizes of wildland fires and reducing the severity of damage from them.

Combining fuelbreaks with area-wide fuel treatments in adjacent areas can reduce the size and intensity of wildland fires. For glades size is 1.5 m fire front spreads only in the wind direction and then goes backwards near the upper glade.
5 Conclusion

The results of the computational experiments illustrate some specific features of the simulated processes of surface forest fires; confirm the variety of dynamics options and different scenarios for fire fronts propagation. It is highly important to continue researches and to clarify the defining variables and functions of the mathematical model.

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