Numerical modeling of the dynamics of the spring thermal bar in Lake Dolgoe (Belarus): preliminary results

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Abstract. The results of numerical modeling of the dynamics of the spring thermal bar into Lake Dolgoe (Belarus), obtained using meteorological data from the Sharkovshchina station for April 2023, are presented. The dependence of the horizontal movement of the thermal bar front on the wind direction is revealed: the northern wind accelerates it, and the eastern wind slows it down. Modeled temperature distributions showed warming of water not only in the area where the channel flows from Lake Svyadovo, but also in the middle shallow part of the body of water.

1 Introduction

Lake Dolgoe is located in the Glubokoe district of the Vitebsk region and is the deepest lake in Belarus. Its depth is 53.7 m. The basin of the lake is of glacial origin, hollow type, stretches for 6 km from northwest to southeast, which coincides with the direction of the prevailing winds. The geographical location of the research object is shown in Figure 1.

Currently, the lake's ecosystem is in conditions close to natural, but until the 1990s. Agriculture actively developed in its catchment area. The activities of the livestock complex on the shores of Lake Dolgoe led to the end of the 1970s. to a significant deterioration of the ecological condition of the body of water. The discharge of untreated wastewater with a high content of nutrients has led to a decrease in water transparency and massive development of phytoplankton. The current ecological state of the body of water is determined by the supply of mineral salts, organic and biogenic compounds from the agricultural lands of the lake's drainage basin. Human impact on the ecosystem of Lake Dolgoe can lead to a reorganization of the trophic structure of the biocenosis, the destruction of rare species of flora and fauna (whitefish and smeltfish ichthyofauna and relics of the Ice Age), and a severe deterioration in the sanitary and hygienic quality of water.

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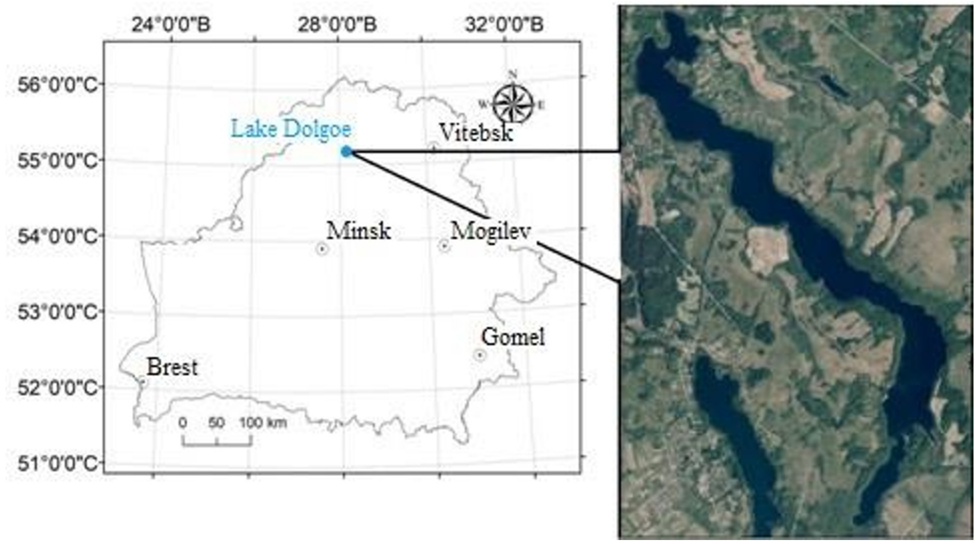

Fig. 1. Geographical location of Lake Dolgoe and the structure of its small catchment area.

Among the natural phenomena that influence the ecological state of a body of water is a thermal bar, which occurs in lakes of temperate latitudes in the spring and autumn seasons and represents a narrow zone of sinking of water in the region of maximum density temperature. The formation of a thermal bar occurs in the coastal part of the body of water, and its destruction occurs in the central part. The development of the spring (autumn) thermal bar is facilitated by the heating (cooling) of the body of water. In spring and autumn, the ecological state of lakes at temperate latitudes directly depends on the activity of the thermal bar, since it creates a barrier to horizontal water exchange, forming temperature differences in separate areas - *thermoactive* in the littoral zone and *thermoinert* in the pelagic zone [1].

The purpose of this work is to reproduce the effect of the spring thermal bar using the example of the bathymetry of Lake Dolgoe based on a 2.5D mathematical model.

2 Materials and methods

2.1 Mathematical model

2.1.1 Basic equations of the model

The non-hydrostatic 2.5D model, which takes into account the influence of the Coriolis force, consists of the following basic equations:

$$\frac{\partial u}{\partial t}+\frac{\partial u^2}{\partial x}+\frac{\partial uw}{\partial z}=-\frac{1}{\rho_0}\frac{\partial p}{\partial x}+\frac{\partial}{\partial x}\left(K_x\frac{\partial u}{\partial x}\right)+\frac{\partial}{\partial z}\left(K_z\frac{\partial u}{\partial z}\right)+2\Omega_z v-2\Omega_y w; \quad (1)$$

$$\frac{\partial v}{\partial t}+\frac{\partial uv}{\partial x}+\frac{\partial wv}{\partial z}=\frac{\partial}{\partial x}\left(K_x\frac{\partial v}{\partial x}\right)+\frac{\partial}{\partial z}\left(K_z\frac{\partial v}{\partial z}\right)+2\Omega_x w-2\Omega_z u; \quad (2)$$

$$\frac{\partial w}{\partial t}+\frac{\partial uw}{\partial x}+\frac{\partial w^2}{\partial z}=-\frac{1}{\rho_0}\frac{\partial p}{\partial z}+\frac{\partial}{\partial x}\left(K_x\frac{\partial w}{\partial x}\right)+\frac{\partial}{\partial z}\left(K_z\frac{\partial w}{\partial z}\right)-\frac{g\rho}{\rho_0}+2\Omega_y u-2\Omega_x v; \quad (3)$$

$$\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}=0; \quad (4)$$

$$\frac{\partial T}{\partial t}+\frac{\partial uT}{\partial x}+\frac{\partial wT}{\partial z}=\frac{\partial}{\partial x}\left(D_x\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial z}\left(D_z\frac{\partial T}{\partial z}\right)+\frac{1}{\rho_0 c_p}\frac{\partial H_{sol}}{\partial z}; \quad (5)$$

$$\frac{\partial S}{\partial t}+\frac{\partial uS}{\partial x}+\frac{\partial wS}{\partial z}=\frac{\partial}{\partial x}\left(D_x\frac{\partial S}{\partial x}\right)+\frac{\partial}{\partial z}\left(D_z\frac{\partial S}{\partial z}\right), \quad (6)$$

where *u, v, w* are the velocity components; T – temperature; S – mineralization; g – acceleration of gravity; D_x (K_x) and D_z (K_z) are the coefficients of turbulent diffusion (viscosity) in the corresponding directions; Ω_x, Ω_y and Ω_z – components of the angular velocity vector of the Earth's rotation; p – pressure; c_p – specific heat capacity; ρ_0 is the density of water at standard atmospheric pressure, characteristic temperature and salinity.

The absorption of solar radiation is calculated using the Bouguer–Lambert–Baer formula:

$$H_{sol}=H_{Ssol,0}\left(1-r_s\right)\exp\left(-\varepsilon_{abs}d\right) \quad (7)$$

where r_s is the coefficient of reflection of water *(≈0.2)*, ε_{abs} is the coefficient of absorption of solar radiation in water *(≈0.3 m^{-1})*, d is depth. The influx of solar radiation onto the water surface $H_{Ssol,0}$ is calculated

$$H_{Ssol,0}=\begin{cases} S_0\cdot\left(a_g-a_w\right)\cdot\cos\zeta\left[a(C)+b(C)\ln\left(\cos\zeta\right)\right], & \cos\zeta>0;\\ 0, & \cos\zeta\le 0,\end{cases} \quad (8)$$

where is the solar constant *(≈1367 W/m^2)*; ζ – zenith angle of the Sun; $a(C)$, $b(C)$, a_g, a_w – empirical coefficients and functions depending on atmospheric parameters.

Closing the system and calculating the water density are carried out using the *k–ω* turbulence model [2] and the Chen–Millero equation of state [3], respectively.

2.1.2 Initial and boundary conditions

Initial conditions for the basic equations of the model:

$$u=0;\ v=0;\ w=0;\ T=T_L;\ S=S_L$$

where T_L and S_L are the temperature and salinity in the lake, respectively.

Boundary conditions:

- At the interface between the body of water and the atmosphere

$$K_z \frac{\partial u}{\partial z} = \frac{\tau^u_{surf}}{\rho_0}; K_z \frac{\partial v}{\partial z} = \frac{\tau^v_{surf}}{\rho_0}; w = 0; D_z \frac{\partial T}{\partial z} = \frac{H_{net}}{\rho_0 \cdot c_p}; \frac{\partial S}{\partial z} = 0.$$

where τ^u_{surf} and τ^v_{surf} are the components of the surface shear stress of the wind, H_{net} is the totality of the fluxes of long-wave radiation (H_{lw}), latent (H_L) and sensible (H_S) heat:

$$H_{lw} = \varepsilon_w \varepsilon_a \sigma \left(1 + 0.17C^2\right) T_A^4 - \varepsilon_w \sigma T^4 \tag{9}$$

where T_A is the air temperature; ε_w and ε_a are the emissivity coefficients of water and atmosphere, respectively; C– cloudiness; σ – Stefan–Boltzmann coefficient (=5.669×10^{-8} W/m^2/K^4);

$$H_L = f_u \left(e_A - e_w\right) \tag{10}$$

where is the mass transfer coefficient; and are the pressure of water vapor in the atmosphere and the pressure of saturated water vapor, respectively;

$$H_S = \beta \cdot f_u \left(T_A - T\right) \tag{11}$$

where β is the Bowen coefficient (=0.62 hPa/K).

- On solid boundaries

$$u = 0; v = 0; w = 0; \frac{\partial T}{\partial n} = 0; \frac{\partial S}{\partial n} = 0$$

where n is the direction of the outer normal to the area;

- At the river-body of water interface

$$u = u_R; \ v = 0; \ w = 0; \ T = T_R; \ S = S_R$$

where u_R is the river speed; T_R and S_R are the temperature and salinity of water in the river, respectively.

- On an open boundary [4]

$$\frac{\partial \varphi}{\partial t} + c_\varphi \frac{\partial \varphi}{\partial x} = 0; \ \frac{\partial w}{\partial x} = 0$$

where $\varphi = u, v, T, S$.

2.1.3 Methods

The problem is solved numerically based on the finite volume method, according to which the components of the velocity vector are determined at the midpoints on the faces of the mesh cell, and the scalar quantities are determined at its center. Approximation of the computational domain to the lake bottom profile is carried out by the method of blocking fictitious domains [5]. To approximate the nonstationary and convective terms in the equations, the Crank–Nicholson and QUICK schemes are used, respectively [6]. To solve difference equations, relaxation and N.I. methods are used. Buleev [7]. The velocity field is matched with the pressure field using the original SIMPLED procedure [8].

3 Study area and task parameters

The longitudinal section of Lake Dolgoe is considered (Figure 2, left), the length and depth of the computational domain (which is covered with a grid with steps h_x=12.5 m and h_z=1.5 m) are 4.6 km and 54 m, respectively (Figure 2, right). Bathymetric data taken from [9]. Time step – 20 s.

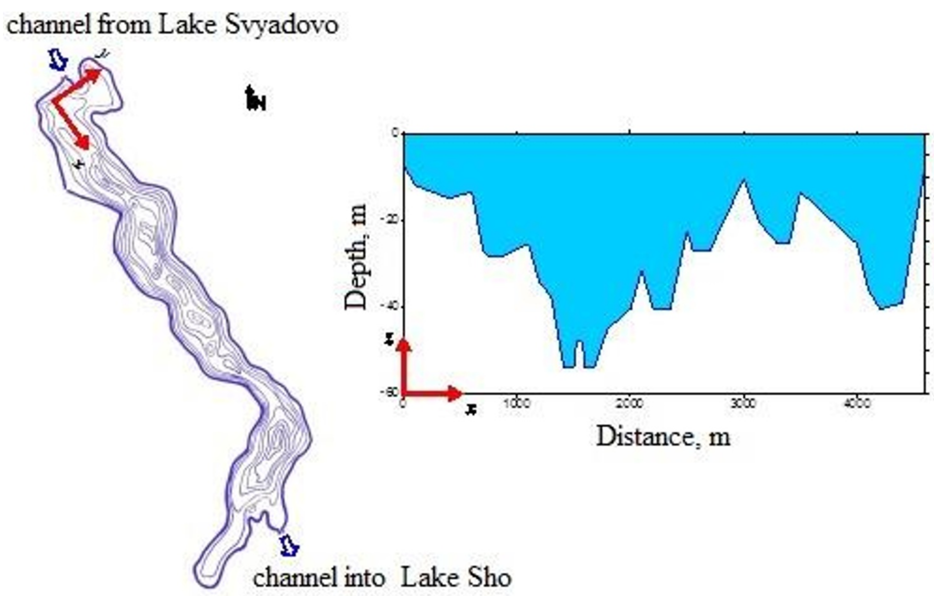

Fig. 2. Lake Dolgoe (left) and computational area (right).

The initial value of water temperature in Lake Dolgoe is taken to be 3.0 °C. The temperature of the water coming from the channel from Lake Svyadovo increases from 3.0 °C to 0.2 °C per day. The mineralization of water in the lake and river is set to constant (200 mg/l). The flow speed from the channel from Lake Svyadovo is 0.2 cm/s. The horizontal viscosity and diffusion coefficients K_x and D_x are assumed to be 1.0 m^2/s. The calculations use meteorological data from the Sharkovshchina station for April 2023 [10]. Wind speed and direction are shown in Figure 3.

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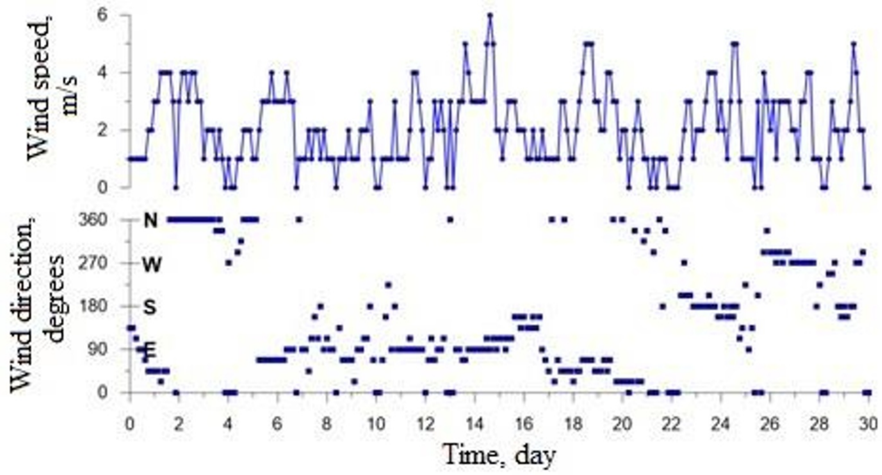

Fig. 3. Wind speed (above) and direction (bottom) in April 2023 according to the weather station in the town of Sharkovshchina.

4 Results and Discussion

The calculated values of heat flows in April 2023 according to the weather station of the Sharkovshchina settlement (Figure 4) varied within the following limits: latent heat from -127.3 to 0 W/m^2, sensible heat from -25.6 to 77.3 W/m^2, long-wave radiation from -120.1 to 37.7 W/m^2 and shortwave radiation from 0 to 628.4 W/m^2.

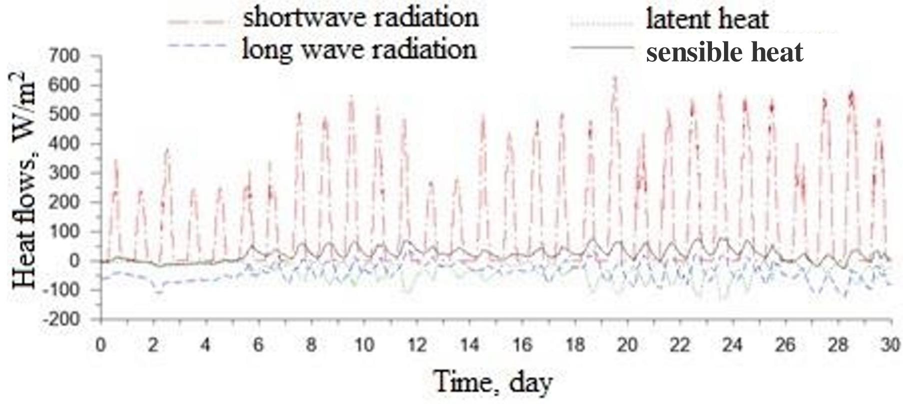

Fig. 4. Heat flows in April 2023.

The isotherms obtained on days 12 and 18 (Figure 4) indicate the heterogeneity of temperature distribution in Lake Dolgoe, which is associated with the bathymetric features of the body of water. Warming is observed not only in the area where water enters from the channel from Lake Svyadovo, but also in the middle, shallow part of the section (at a distance of 3000 - 3500 m). However, a noticeable increase in temperature occurs due to the influx of warmer channel waters from Lake Svyadovo (maximum values on days 12 and 18 were 4.7 °C and 5.9 °C, respectively). Minimum values from 12 to 18 days increased from 3.4 °C to 3.7 °C. During this time, the thermal bar front moved from 600 m from the mouth to 1250 m, and one can note the vertical nature of the maximum density temperature profile both on the 12th day and on the 18th day (Figure 5).

From the dynamics of movement of the area where the temperature of maximum density is located (Figure 6) it is clear that during the warming up of the body of water, the thermal bar front did not move very actively and often even changed the direction of movement towards the mouth, and this can be associated with the predominance of eastern winds on days 9-20 (Figure 3). So on days 9-11 the wind direction varied mainly between east and north. The northern wind contributed to the faster movement of the thermal bar, and the eastern one slowed down its development, but the wind speed during this period of time did not exceed 3 m/s, and therefore its influence was not so great. On the 12-15th day, the wind blew predominantly from the east and close directions at a speed of up to 4-6 m/s, significantly slowing down the development of the thermal bar. On days 16-18, the wind speed dropped again to 3 m/s, and the direction began to have a greater scatter, which generally led to a smaller slowdown in the movement of the thermal bar into the central part of the lake. On days 19 and 20, strong wind appears again at a speed of up to 5 m/s, which increases the role of wind during this period of time. On the 19th day, the wind direction is closer to the east-northeast and northeast, and on the 20th day - to the north-northeast and north, which was reflected in some stagnation in the temperature of the maximum density on the 19th day and its active advancement on the 20th day .

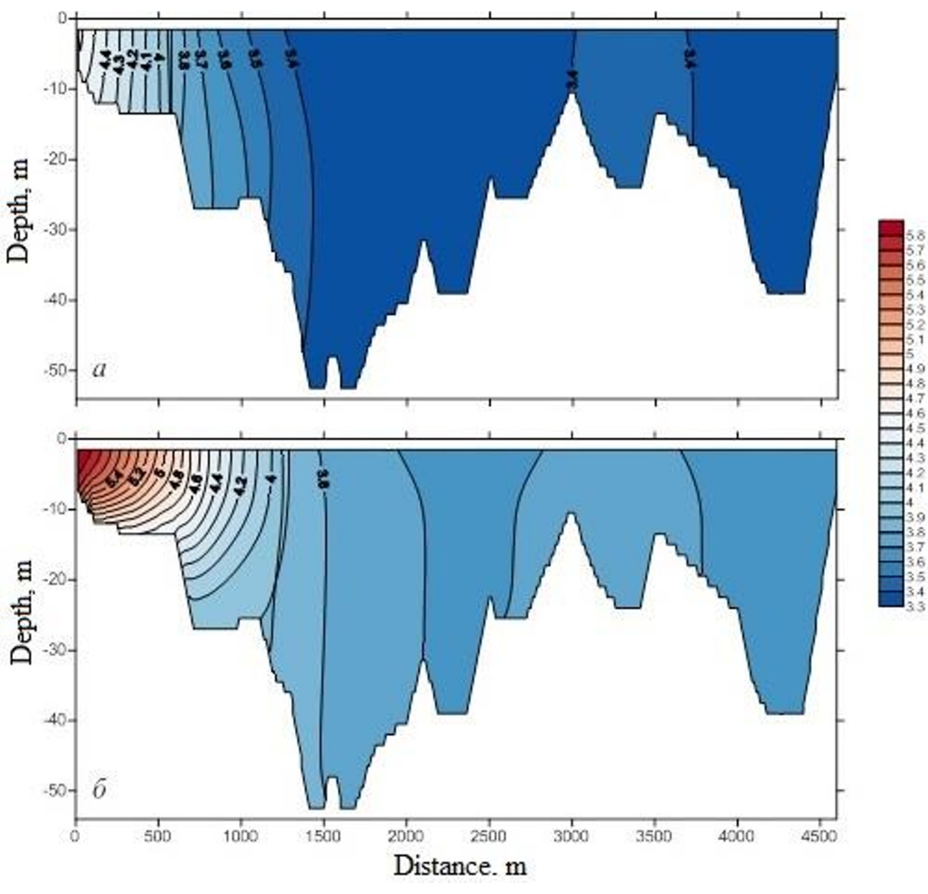

Fig. 5. Temperature distribution in Lake Dolgoe after 12 (a) and 18 (b) days. The thick line shows the temperature profile of maximum density.

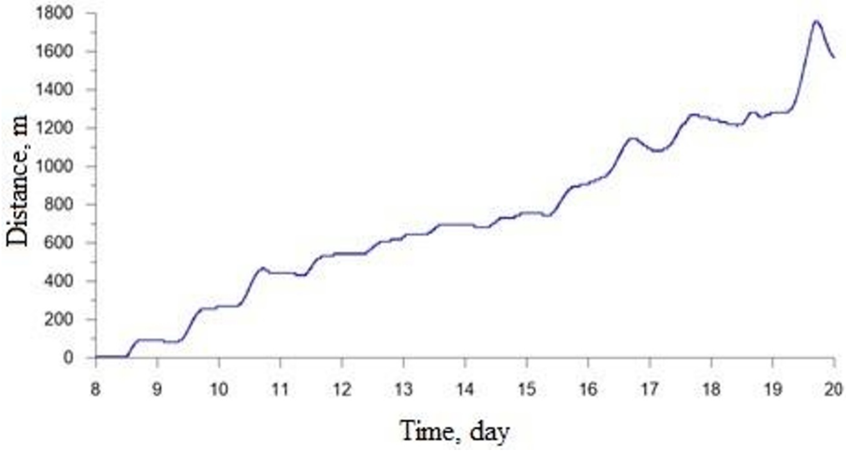

Fig. 6. Dynamics of movement of the region where the temperature of maximum density is located.

5 Conclusion

Preliminary results of numerical modeling of thermohydrodynamic processes in Lake Dolgoe showed uneven heating of water in the lake due to the morphometric features of the basin. According to the obtained temperature profiles of maximum density, the front of the thermal bar into Lake Dolgoe passes from the surface to the bottom. In the future, work will be carried out to clarify the initial and boundary conditions for the numerical model after carrying out spring field measurements of physical parameters.

Acknowledgement

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