## MATHEMATICAL SIMULATION IN OPHTHALMOLOGY

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Shell-like structures are of very frequent occurrence in biological system, particularly, in a human eye. Shell theory-based mechanical models have been used in recent years as tools to describe, for example, the stress-strain state of the eye shell under encircling band; to build a biomechanical model of the choroidal detachment, to depict the different mechanical aspects of the development of glaucomatous atrophy of the optic nerve fibers and the behavior of Lamina Cribrosa – circular or closed to circular plate, where the optic nerve fibers pass through [1].

A key diagnostic parameter to determine the health of the eye is the fluid pressure inside an eye called the intraocular pressure (IOP). Last years a number of works devoted to study the parameter of ocular rigidity which connects with relation «IOP – volume of eye» are appeared. In clinical practice IOP is measured through the corneoscleral coat by different types of tonometers. Due to new types of refractive surgery the problem of the IOP measurements standardization, the assessment of the effect of individual variations in geometrical and mechanical parameters of the eyeball on the accuracy of IOP readings have acquired special attention. Thus, developing the adequate mathematical models for tonometry is highly important problem. The effect of the refractive surgery on the IOP readings can be evaluate by comparing the simulation results with clinical data before and after surgery.

Intravitreal injections are extensively used to treat retinal diseases nowdays. Estimation of the short time effect of intravitreal injections on the IOP elevation is another possible application of shell-theory based models. It is necessary to take into account structural features of eyes in order to obtain results close to clinical data. The outer coat of eye could be considered as three-layered shells – the sclera, the choroid and the retina, respectively. The mechanical and geometrical characteristics of the layers are different. In the first approximation sclera, choroid and retina could be considered as transversal-isotropic shells. It is known also that mechanical properties of sclera are different for eyes with normal vision, for eyes with myopia or hyperopia. Form of eyes with normal vision is closed to sphere, eyes with myopia and hyperopia often have ellipsoidal forms.

If we consider the stress-strain state of isotropic spherical layer under normal pressure then the solution could be obtained with the exact 3D theory [2]. For transversal-isotropic layer the same problem also could be resolved in 3D theory [3]. It is interesting to compare the results, which found with 3D theory for multilayered spherical shells with results obtained with refined theories for orthotropic shells of moderate thickness by Paliy-Spiro (PS) [4] and by Rodionova-Titaeva-Chernykh (RTC) [5] and also to compare results for three-layer shell with those for one layer with average elastic properties.

For transversally isotropic spherical shell of one layer the displacement of the middle surface by means of the 2D PS theory ($u^{\text{PS}}$) and the RTC theory ($u^{\text{RTC}}$)

could be represented as

$$u^{\mathrm{PS}}/u^{\mathrm{KL}} = 1-\alpha\left(1-\nu^{*}\right),$$

$$\frac{u^{\mathrm{RTCh}}}{u^{\mathrm{KL}}} = 1-\alpha\left(1-\nu^{*}\right)+\frac{\alpha^{2}}{12}\left(3-12\nu^{*}-2\left(\nu^{*}\right)^{2}-\frac{6E_{1}\left(1-2\nu'\nu^{*}\right)}{5(1-\nu)E_{3}}\right),$$

where $u^{\mathrm{KL}}$ is Kirchhoff-Love approximation of the mid-surface deflection $u^{\mathrm{KL}} = \frac{(1-\nu)\,pR^{2}}{2E_{1}h}$, $p$ – the load parameter, $\alpha = \frac{p}{K}$ – the relative shell thickness, ($h$ is the shell thickness and $R$ is the typical radius of the curvature), $E_1$, $E_3$, $\nu$, $\nu'$ – the independent elastic parameters,

$$\nu^{*} = \frac{E_{1}}{E_{3}}\frac{\nu'}{1-\nu}.$$

If α<<1, then first members of the asymptotic expansion of the exact 3D – theory ($u^{\mathrm{3D}}$) gives [6]

$$\frac{u^{\mathrm{3D}}}{u^{\mathrm{KL}}} = 1-\alpha\left(1-\nu^{*}\right)-\alpha^{2}\,\frac{3\nu'-\nu^{*}-11\nu'\nu^{\wedge}}{12\nu'}+O\left(\alpha^{3}\right).$$

For normal stresses on the middle surface we obtain

$$\frac{\sigma_{zz}^{\mathrm{PS}}}{p} = -\frac{1}{2}\left(1-\alpha-\frac{\alpha^{2}}{4}\right),\quad \sigma_{zz}^{\mathrm{RTC}} = -\frac{1}{2}\left(1-\frac{3}{2}\alpha+\frac{3\alpha^{2}}{8}\right),$$

$$\frac{\sigma_{zz}^{\mathrm{3D}}}{p} = -\frac{1}{2}\left(1-\alpha-\frac{\alpha^{2}}{4}\left(\frac{E_{1}}{E_{3}}\frac{1-\nu'}{1-\nu}-1\right)\right)+O\left(\alpha^{3}\right).$$

For multilayered shells the similar, but much lengthy formulas are obtained.

The comparison of obtained relations shows that both theories of shells give two first terms of 3D solutions for the displacement of the middle surface. For normal stresses on the middle surface the theory of Paliy-Spiro gives more exact result.

In the multilayer shell circumferential stresses possessed jump discontinuities near layer-to layer contact lines. For the eyeball such effect could cause the detachments of the inner layers of the eye shell under increased intraocular pressure.

In conclusions we can say that both theories (PS and RTC) can be applied to evaluate the increasing of intraocular pressure after intravitreal injection in eyes with myopia or hyperopia, i.e. eyes of an ellipsoidal shape.

**References**


1. Bauer S. M., Zimin B. A., Tovstik P. E. *Simplest Models of Plates and Shells in Ophthalmologys.* St. Petersburg, SPbGU – 2000 – 92 p. (in Russian)
2. Atanackovic T. M., Guran A. *Theory of Elasticity for Scientist and Engineers*. Birkhauser-Berlin. – 374 p.

3. Bauer S. M., Kotliar K. E., Maier M., Feucht N., Lohmann C., Lanzl I. *Effect of Intravitreal Injections and volume changes on Intraocular Pressure: clinical results and biomechanical model* // Acta Ophthamologica Scandinavica, 2007, No. 85 (7) – P. 777 – 781.
4. Paliy O. M., Spiro V. E. *Anisotropic Shells in Shipbuildings. Theory and Analysis.* – Sudostroenie, 1977. – 392p. (in Russian)
5. Rodionova V. A., Titaev V. F., Chernykh K. F. *Applied Theory of Anisotropic Plates and Shells.* – St. Petersburg University Press, St. Petersburg, 1996 – 278 p. (in Russian)
6. Bauer S. M., Voronkova E. B. *Nonclassiacl shell theories for the analysis of transversally isotropic spherical and cylindrical layers under normal pressure* // Vestnik SPbGU, – 2011, No. 3, P. 86 – 93 (in Russian)

## EXPERIMENTAL METHODS OF MICRO-AND NANOMECHANICS

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Modern technologies are related with the miniaturization trend in technic. Design of micro-and nanoelectromechanical systems (MEMS and NEMS), the creation of nanostructures materials requires an assessment of the local mechanical properties of thin films and materials in nanovolumes (nanoscale clusters, particles, thin coatings, nanocomposites). Commonly used reference characteristics of physical and mechanical properties of materials relevant to their block states and may differ from the properties at the nanoscale. At the same time the systematic values of physical and mechanical properties for the different classes of materials at the nanoscale are absent. In this connection, the role of experimental methods nanomeasurements cannot be overestimate. And mechanic got an ultramodern sounding in the context of the development of nanotechnology. In the field of nanotechnologies, it occupies a privileged position as one of the main methods in nanoworld investigations - scanning probe microscopy is based on the principles of mechanics of contact and non- contact interaction of micro tip with the test object.

Experimental studies confirm the efficiency of the continuum mechanics laws at nanoscale, for example, when we want to interpret the data of nanoindentation in the elastic range of materials. Therefore, in task of experimental nanomechanics the methods of continuum mechanics and molecular dynamics model, taking into account the discrete nature of the materials are used.

In this report we try to represent the current level of experimental methods development for nanomechanics, primarily based on scanning probe microscopy, including the achievements of Belarusian scientists.

Scanning probe microscopy combines a wide class of investigation methods of single nano-objects and surfaces using mechanical scanning tip (probe). Scanning probe microscopy gives an opportunity to evaluate the physical and mechanical properties of materials with a high resolution, up to a few nanometers, and even